Physics of the cardiovascular system
each contraction pumps ≈75 - 80 ml of blood
⇒ fraction of: $t \approx \frac{0.04}{4.4} \approx 0.018$ contraction (heart beat)

With an average heart beat frequency of 1 Hz:
⇒ timescale of $\approx 1$ min/circulation.
Major components of the cardiovascular system

• 1-Pulmonary circulation
• 2-Systemic circulation in the rest of the body
• The blood is pumped by the contraction of the heart muscles from the left ventricle at a pressure of about 125mmHg into a system of arteries that subdivided into smaller arteries (arterioles) and to capillary bed. During the a few seconds it is in the capillary bed the blood supplies O2 to the cells and picks up CO2 from the cells.
After passing through the capillary bed the blood collects in small veins that gradually combine into larger veins before entering the right side of the heart. The returning blood is momentarily stored in the reservoir (the right atrium), and during a weak contraction (5 to 6mmHg) the blood flows into right ventricle. On the next ventricular contraction this blood is pumped at a pressure of about 25mmHg. Via the pulmonary arteries to the capillary system in the lungs, where it receives more O2 and where some of the CO2 diffuses into the air in the lungs to be exhaled. The freshly oxygenated left reservoir of the heart (left atrium) during weak contraction (7 to 8mmHg), the blood flows into the left ventricle. On the next ventricular contraction this blood is again pumped from the left side of the heart into the general circulation.
O2 and CO2 exchange in the capillary system
Oxygen and carbon dioxide diffuse through tissue due to the low

\[ D = \lambda \sqrt{N} \]

\( D \) is the most probable distance

\( N \) is number of collisions

\( \lambda \) is the average distance between collisions
Blood pressure and its measurement

The blood pressure is measured by sphygmomanometer. The arterial blood flow to the arm is blocked by an inflated cuff. As the air gradually released, the stethoscope placed over the brachial artery is used to listen for korotkoff sounds. The pressure at which the sound appear and change are noted on the gauge.
The pressure varies from one point to another because of gravitational forces. The greater pressure $P$ in the foot is due to the gravitational force $(\rho g h)$ produced by the column of blood (of height $h$) between the heart and foot added to the pressure at the heart, similarly, the decreased pressure in the head is due to the elevation of the head over the heart.
If gravity on earth suddenly became three times greater, blood would rise only about 4.3cm above the heart and it would not reach the brain of a standing person. This situation can be produced artificially by acceleration the body at 3g in vertical direction. This condition produces pooling of blood in the legs.
Pressure across the blood vessel wall

The greatest pressure drop in the cardiovascular system occurs in the region of the arterioles and capillaries. The capillaries have very thin walls that permit easy diffusion of $O_2$ and $CO_2$. In order to understand why they do not burst we must discuss the wall of a tube which is related to the radius of the tube and the pressure inside the tube:

$$T = RP$$
Consider along tube of radius R carrying blood at pressure P. We calculate the tension T in the wall. We can divide the tube in half as shown in figure.

The force per unit length pushing upward is 2RP. There is a tension force T per unit length at each edge that holds the top half of the tube to bottom half. Since the wall is in equilibrium the force pushing the two halves apart is equal to the tension forces holding them to gather or \(2T = 2RP\) or \(T = RP\)
For very small radius the tension is also very small.

**Typical pressure and tensions in blood vessel**

<table>
<thead>
<tr>
<th></th>
<th>P(mmHg)</th>
<th>R(cm)</th>
<th>T(dyne/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aorta</td>
<td>100</td>
<td>1.2</td>
<td>156000</td>
</tr>
<tr>
<td>Typical artery</td>
<td>90</td>
<td>0.5</td>
<td>60000</td>
</tr>
<tr>
<td>Small capillary</td>
<td>30</td>
<td>6x10^{-4}</td>
<td>24</td>
</tr>
<tr>
<td>Small vein</td>
<td>15</td>
<td>2x10^{-2}</td>
<td>400</td>
</tr>
<tr>
<td>Vena cava</td>
<td>10</td>
<td>1.5</td>
<td>200000</td>
</tr>
</tbody>
</table>

The tension in the wall of aorta is about 156000 while the tension in a capillary wall is 24 dyne/cm.
Bernoulli's principle applied to the cardiovascular system

Energy per unit volume before = Energy per unit volume after

\[ P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2 \]

Pressure Energy  Potential Energy per unit volume  Kinetic Energy per unit volume

Flow velocity \( v_1 \)  Flow velocity \( v_2 \)

The often cited example of the Bernoulli Equation or "Bernoulli Effect" is the reduction in pressure which occurs when the fluid speed increases.

\[ A_2 < A_1 \]
\[ v_2 > v_1 \]
\[ P_2 < P_1 \]

Increased fluid speed, decreased internal pressure.
Bernoulli's principle is based on the law of conservation of energy. Pressure in fluid is a form of potential energy \( PE \) since it has ability to perform useful work.

In a moving fluid is kinetic energy \( KE \) due to motion. Kinetic energy can be expressed as energy per unit volume.
If fluid is flowing through the frictionless tube shown in figure, the velocity increases in the narrow section and increased in kinetic energy KE of the fluid is obtained by a reduction of the potential energy of the pressure in the tube. As the velocity reduces again on the far side of the restriction the kinetic energy is converted back into potential energy and the pressure increases again as indicated on the manometers.

We can calculate the average kinetic energy of blood as leaves the heart.

\[ \text{KE} = \frac{1}{2}mv^2 \]  since average velocity = 30 cm/s

\[ \text{KE} = \frac{1}{2} \times 30^2 = 450 \text{ ergs/cm}^3 \text{ or } 450 \text{ ergs} \]

This kinetic energy is equivalent to a potential energy 450 dyne/cm², since a pressure of 1 mmHg corresponds to 1330 dyne/cm², this potential energy amounts to less than 0.4 mmHg.
Relating the Continuity of flow equation \((A_1v_1 = A_2v_2)\) with Bernoulli’s equation. Continuity of flow equation tells us this: when the area decreases, the velocity increases in order to maintain a constant flow rate.
Bernoulli’s equation tells us that when velocity increases, the pressure (that the fluid exerts on its walls) decreases.
When you have cholesterol buildup and arterosclerosis, then the arteries decrease in area since the radius is smaller. From the continuity of flow equation, the velocity of the blood must increase to maintain the same flow rate \(Q\). This increase in velocity results in a lower pressure at that area.

\[ p + \frac{1}{2} \rho V^2 + \rho gh = \text{constant} \]

where \(p\) is the pressure, \(\rho\) is the density, \(V\) is the velocity, \(h\) is elevation, and \(g\) is the gravitational acceleration.
However, the arteriosclerosis would cause a decrease in R in the equation, and consequently cause an increase in the pressure gradient in order to maintain the same flow rate. This means that the heart should increase the pressure (high blood pressure as is observed).
Hypertension

High blood pressure (hypertension) is generally caused by factors such as the renin-angiotensin system, salt and water retention etc... Contrary to popular belief hypertension is usually a cause of cholesterol deposits rather than being caused by these. Hypertension damages the artery walls and allows atheromatous plaques to form. Once cholesterol has been deposited in an artery wall perfusion is reduced for a given level of blood pressure:
From continuity and Bernoulli’s equations

From the continuity equation, $A_1 v_1 = A_2 v_2$.

Due to cholesterol deposits, $A_1 > A_2 \rightarrow v_1 < v_2$.

From Bernoulli’s equation, $\frac{1}{2} \rho v_2^2 + \rho gh + P = \text{constant}$.

Thus, since $v_1 \downarrow \rightarrow P_1 \uparrow$
How fast does your blood flow

• The blood goes from the aorta into the smaller arteries and arterioles with greater total-cross-sectional areas the velocity of the blood decreases

• The velocity = \( \frac{\text{flow rate}}{\text{cross-sectional}} \)

• The average velocity in the aorta 30cm/s; that in a capillary is only about 1mm/s, this low velocity allows time for diffusion of gases to occur.
The flow rate due to Poiseuilles law = $\Delta P \left( \frac{\pi}{8} \right) \left( \frac{1}{\eta} \right) \left( \frac{R^4}{L} \right)$

- The factors affect the flows of blood in the vessel are.
- 1-viscosity ($\eta$), the cgs unit used to measure viscosity is Poise, the SI unit for viscosity is Pascal second (Pas), which equals 10 poises. The viscosity of blood is typically $3 \times 10^{-3}$ to $4 \times 10^{-4}$ pas.
2- Pressure difference ($\Delta P$). If $\Delta P$ is doubled, the flow rate also doubles.

3- The length ($L$), the flow varies inversely with the length

4- The radius ($R$), if the radius is doubled the flow rate increases by $2^4$ or factor 16.
An important characteristic of laminar flow is that it is silent, if all blood flow were laminar, information could not be obtained from the heart with a stethoscope. The heart sounds heard with a stethoscope are caused by turbulent flow.
If you gradually increase the velocity of fluid flowing in a tube by reducing the radius of the tube, it will reach a critical velocity $V_C$ when laminar flow changes into turbulent:

$$v_C = \frac{k \eta}{\rho R}$$

h: viscosity
R: radius
$\rho$: density

$K$: reynolds number=1000 for many fluids including blood, flowing in tubes of constant diameter. If there are bends the reynolds number becomes much smaller.
In aorta, which has a radius of about 1cm in adult

The \( V_c = \frac{K \eta}{\rho R} = \frac{1000 \times (4 \times 10^{-3} \text{ pas})}{(10^3 \text{ kg/m}^3) (10^{-2} \text{ m})} = 0.4 \text{ m/sec} \)

The velocity in the aorta range from 0 to 0.5 m/s, and thus the flow is turbulent during part of the systole.
Laminar flow is more efficient than turbulent flow. This is illustrated graphically in figure a. The slope of the curve in the laminar flow region is greater than that in the turbulent flow region. That is, a given increase in pressure causes a greater increase in the laminar flow rate than in the turbulent flow rate. The reduction in efficiency is apparent in the blood flow through an artery with obstruction (Figure b). For the flow rate \( V_A \) a pressure of \( P_1 \) is needed for the normal artery and somewhat higher pressure \( P_2 \) is needed for the obstructed artery. If both arteries are required to deliver a new flow rate \( V_B \), the increase in pressure \( \Delta P_2 \) will be much greater for obstructed artery since the flow will be turbulent.
Work done by the heart

Each contraction of the heart muscles forces about 80ml of blood through the lungs from the right ventricle and a similar volume from the left ventricle. In the process the heart does work.

The pressure in two pumps of the heart are not the same. In the pulmonary system the pressure is low because of low resistance of the blood vessels in the lungs. The maximum pressure (systole) about 25mmHg. In order to circulate the blood through the much larger systemic net work the left side of the heart muscle produce pressure about 120mmHg at the peak (systole) of each cardiac cycle.
The work $W$ done by a pump working at a constant pressure $P$ is equal to the product of pressure and volume pumped $\Delta V$ or

$$W = P \Delta V$$

Average $P = 10\text{cmHg}$

$$\Delta V = 80\text{ml}$$

$$W = \rho gh \Delta V$$

$$= 13.6 \text{g/cm}^3 \times 980 \text{cm/s}^2 \times 10\text{cm} \times 80\text{cm}^3 = 1.1 \times 10^7 \text{ ergs}$$

Or $1.1 \text{ J}$ or $1.1 \text{ J/s}$ or a power of $1.1\text{W}$
The physics of some cardiovascular diseases

Heart diseases often have physical component. Many of these diseases, for example, increase the work load of the heart or reduce its ability to work at a normal rate.

\[ W = P \Delta V \] equation of work done

\[ T = P R \] Laplace equation
The work done by the heart is roughly the tension of the heart muscle times how long it acts. Anything that increase the work load of the heart. For example 1- Hypertension

Causes the muscle tension to increase in proportion to the pressure, due to Laplace law $T = PR$ so the high blood pressure causes to increase the work done by equation $W = P\Delta V$. 
2- Tachycardia

A fast heart rate increases the work load since the amount of time the heart muscle spend contracting increases
3-Enlargement of the heart and reduction in the ability of the heart to provide adequate circulation.

If the radius of the heart is doubled, the tension of the heart muscle must also be doubled if the same pressure is to be maintained. Since the heart muscle is stretched, it may not be able to produce sufficient force to maintain normal circulation, the stretched heart muscle is also much less efficient than normal heart muscle.